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GEOMETRY.

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389. Proposed by H. PRIME, Boston, Mass.

On the same side of a given base, triangles are erected such that the bisectors of their vertex angles all pass through a given point. Find the locus of the vertices (i) when the vertex angle are all equal, (ii) when the vertex angles are all unequal.

Solution by the PROPOSER.

Take the given base $BB=2m$ for the axis of x, its midpoint O for the origin. Let the triangles be erected on the positive side of BB; a, b be the given point, P any point of the required locus, and the circle BPB, radius r, cut OY' in Q. The coördinates of Q are 0, $\pm\sqrt{(r^2-m^2)}-r$. The angle bisector passes through Q and a, b. Its equation is

$$y=[b\mp\sqrt{(r^2-m^2)}+r]x/a\pm\sqrt{(r^2-m^2)}-r\ldots(1).$$

The equation of the circle BQB is

$$x^2+[y\mp\sqrt{(r^2-m^2)}]=r^2\ldots(2).$$

The upper sign apply when the center of the circle is above the axis of x, the lower sign when it is below.

Eliminating the variable parameter r between (1) and (2), we have

$$\text{(A)}\quad 2ay^2+2bxy+(a-x)(x^2+y^2-m^2\mp\sqrt{[(x^2+y^2-m^2)^2+4m^2y^2]},$$
$$\text{or, } y(ay-bx)^2-(a-x)(x^2+y^2-m^2)(ay-bx)=m^2y(a-x)^2.$$

This is the general equation of the locus and defines its form in every relation of the constants. If $a=0$, (A) reduces to either

$$\text{(B)}\qquad x=0, \text{ or,}\qquad\text{(C)}\qquad x^2+y^2-y(b^2-m^2)/b=m^2.$$

(B) applies when the triangles are isosceles, their vertex angles unequal and bisected by the axis of y.

(C) is the equation of a circle whose radius$=(b^2+m^2)/2b=$a constant. Hence all the triangles are inscribed in the same circle and, having the same base, their vertex angles are equal.

Also solved by C. N. Schmall.

390. Proposed by PROF. R. C. ARCHIBALD, Brown University, Providence, R. I.

Find, geometrically and without introducing focal properties, the locus of the vertices of the conjugate parallelograms of an ellipse.

Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

Let $y=mx+\sqrt{(a^2m^2+b^2)}$, $y=m'x+\sqrt{(a^2m'^2+b^2)}$ be the equation of the tangents parallel to two conjugate diameters. But between m and m' there is the well known relation: $mm'=-(b^2/a^2)$, whence $m'=-(b^2/a^2m)$. Substituting this in the second of the above equations, we have $a^2ym+b^2x=ab\sqrt{(a^2m^2+b^2)}$, and dividing the last equation by $y-mx=ab\sqrt{(a^2m^2+b^2)}$, we obtain

$$m=\frac{b}{a}\cdot\frac{ay-bx}{ax+bx};$$

and substituting in the latter equation, we finally get $a^2y^2+b^2x^2=2a^2b^2$, the equation of a concentric ellipse with the semi-axes $a\sqrt{2}$ and $b\sqrt{2}$.

391. Proposed by W. J. GREENSTREET, M. A., Editor, The Mathematical Gazette, Burghfield, England.

An ellipse is inscribed in the triangle of reference and has one focus at $(\sec A, \sec B, \sec C)$. Find the other focus and the sum of the squares of the axes of the ellipse.

Solution by WILLIAM HOOVER, Ph. D., Athens, Ohio.

In trilinear coördinates, let $(\alpha_1, \beta_1, \gamma_1)$, $(\alpha_2, \beta_2, \gamma_2)$ be the two foci. Then the first is found by

$$\alpha_1\cos A=\beta_1\cos B=\gamma_1\cos C\ldots(1),$$

$$\text{and } a\alpha_1+b\beta_1+c\gamma_1=2\triangle\ldots(2),$$

$$\text{or, } \alpha_1=2R\cos B\cos C,\ \beta_1=2R\cos A\cos C, \text{ and } \gamma_1=2R\cos A\cos B\ldots(3),$$

R being the radius of the circum-circle.

If b_1 be the semi-minor axis of the ellipse,

$$\alpha_1\alpha_2=b_1^2=\beta_1\beta_2=\gamma_1\gamma_2\ldots(4), \text{ or,}$$

$$\cos B\cos C.\alpha_2=\cos A\cos C.\beta_2=\cos A\cos B.\gamma_2\ldots(5),$$

which with (3) gives

$$\alpha_2=R\cos A,\ \beta_2=R\cos B,\ \gamma_2=R\cos C\ldots(6).$$

(4) now gives $b_1^2=2R^2\cos A\cos B\cos C\ldots(7)$.

Also, d=the distance between the ortho-center and circum-center, is given by

$$d^2=R^2(1-8\cos A\cos B\cos C)\ldots(8).$$